Dynamics theory of deformable solids with quasiparticle excitations in the presence of electromagnetic fields

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Abstract

A full selfconsistent set of equations is deduced to describe the kinetics and dynamics of charged quasiparticles (electrons, holes etc.) with arbitrary dispersion law in crystalline solids subjected to time-varying deformations. The set proposed unifies the nonlinear elasticity theory equation, a kinetic equation for quasiparticle excitations and Maxwell's equations supplemented by the constitute relations. The kinetic equation used [1,2] is valid for the whole Brillouin zone. It is compatible with the requirement for periodicity in k-space and contains an essential new term compared to the traditional form of the Boltzmann equation. The theory is exact in the frame of the quasiparticle approach and can be applied to metals, semiconductors, as well as to other crystalline solids including quantum crystals and low-dimensional lattice structures.

I. INTRODUCTION

There are two fundamental problems when dealing with quasiparticles in crystalline structures. The first one is related to the fact that the quasimomentum $\mathbf{k}$ and the dispersion law $\varepsilon(\mathbf{k})$ of a quasiparticle are well defined only in an ideal periodic lattice. In such a lattice the dispersion law as well as all other physical quantities are periodic functions in the reciprocal space (k-space). However, in any real system the crystal lattice is deformed (e.g. by impurities, elastic deformations, external fields etc.). The most complicated problems concern the quasiparticle dynamics in a time-varying deformed crystal lattice. In an exact description all physical characteristics of a quasiparticle have to be periodic functions of the quasimomentum with periods which are functions of the coordinates and the time. This leads to a dependence of the Brillouin zone boundaries not only on the deformation at a given instant, but also on the local lattice velocity [1,2].

As for the stationary (or quasistationary) case, this difficulty has usually been passed over by introducing a local dispersion law $\varepsilon(\mathbf{k}, u_{ik})$ and further expansion in powers of the small deformation tensor components u_{ik}:

$$\varepsilon(\mathbf{k}, u_{ik}) = \varepsilon_0(\mathbf{k}) + \lambda_{ik}(\mathbf{k}) u_{ik} \tag{1}$$

where λ_{ik} are the deformation potential components,

$$u_{ik}(\mathbf{r}) = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_k} + \frac{\partial x_i}{\partial x_k}\right), \tag{2}$$

and $\mathbf{u}$ is the deformation vector. Such an approach (known as the local lattice approach) is based on the assumption that the deformations are small and smooth functions of position and time and possesses all shortcomings of any linearized theory. In addition, the kinetic equation becomes incompatible with the requirement of the periodicity. To avoid any misunderstandings, note that Eqs.(1) and (2) are written in the co-moving frame.

The second problem is related to the fact that the crystal lattice plays the role of a privileged coordinate frame and no Galilean transformations for quasiparticle characteristics exist. The lack of transformation laws for such quantities as energy, Hamiltonian and

quasimomentum means, in fact, that there is no consistent quasiparticle mechanics. The most fundamental quantity, the dispersion law, is known in a co-moving frame attached to the lattice and this frame is even not inertial at time-varying deformations. On the other hand, all fundamental physical equations, such as conservation laws, variational principles, kinetic eqs. etc. take place in the laboratory frame (L-system). However the concept of dispersion law does not exist in L-system. Hence, in principle, even if the mechanics equations for a quasiparticle were known in C-system, they remain unknown in L-system.

These problems are so old as the quasiparticle approach itself is. They are well known, for example, in the theory of metals where many attempts have been made to derive a complete system of dynamic equations, consisting of equations from the theory of elasticity, a kinetic equation for the electron gas and Maxwell's equations (cf. Refs. [3,4] and the bibliography cited there).

The attempts to deduce the equations of motion for a quasiparticle in L-system can be divided into two groups corresponding to the twofold role of the dispersion law. In the co-moving frame it coincides with both energy and Hamiltonian. Therefore transformations typical of energy and Hamilton function have been proved.

When considering $\varepsilon(\mathbf{k}, u_{ik})$ as Hamiltonian a transformation by a substitutional function of the form

$$\tilde{\Phi}(\mathbf{r}', \mathbf{p}, t) = (\mathbf{r}' + \mathbf{u}(\mathbf{r}', t))\mathbf{p} \tag{3}$$

has been used and the following relations have been obtained (see e.g. [3,4]):

$$\mathbf{k} = \mathbf{p} + \frac{\partial}{\partial \mathbf{r}'}(\mathbf{up}), \tilde{H}(\mathbf{p}, \mathbf{r}, t) = \tilde{\varepsilon}(\mathbf{k}, \mathbf{r}', t) + \dot{\mathbf{u}}\mathbf{p},$$

where $\mathbf{r} = \mathbf{r}' + \mathbf{u}$ are the coordinates in L-system and $\mathbf{p}$ is supposed to be the corresponding quasimomentum. However, $\tilde{\Phi}$ does not depend on the bare mass, m, of the quasiparticle and therefore does not take into account any inertial effects (e.g. the Stewart-Tolman effect in metals, centrifugal forces in rotating bodies etc).

If $\varepsilon(\mathbf{k}, u_{ik})$ is considered as quasiparticle energy then the Galilean transformation applies

as a consequence of the requirement that energy density $\int \varepsilon d^3k$ (as a macroscopic quantity) has to obey Galilean transformations. This yields

$$\tilde{\mathcal{E}} = \varepsilon_0(\mathbf{k}) + \lambda_{ik}(\mathbf{k})u_{ik} + m\dot{\mathbf{u}}\frac{\partial\varepsilon_0}{\partial\mathbf{k}}. \tag{4}$$

It has been shown by many authors that such a transformation is incompatible with the Boltzmann equation. That is why some artificial ways have been used the most successful being that of Landau (cf the footnote in Ref. [3]). He has suggested that (in order to take into account the noninertial properties of the lattice frame) one has first to add to the dispersion law (1) the term $-m\dot{\mathbf{u}}\frac{\partial\varepsilon}{\partial\mathbf{k}}$, setting

$$\varepsilon(\mathbf{k}, u_{ik}) = \varepsilon_0(\mathbf{k}) + \lambda_{ik}(\mathbf{k})u_{ik} - m\dot{\mathbf{u}}\frac{\partial\varepsilon}{\partial\mathbf{k}}. \tag{5}$$

and then to apply the transformation by the substitutional function (3). The result is:

$$\tilde{H}(\mathbf{p}, \mathbf{r}, t) = \varepsilon_0(\mathbf{k}) + \lambda_{ik}(\mathbf{k})u_{ik} + \left(\mathbf{p} - m\frac{\partial\varepsilon}{\partial\mathbf{k}}\right)\dot{\mathbf{u}}. \tag{6}$$

This procedure has been used in the most of recent works. However it cannot be well-grounded due to its internal inconsistency. In fact, if we consider the expression (5) as energy, and take a constant velocity $\dot{\mathbf{u}}$ then we come to the wrong conclusion that the energy in an inertial frame could depend on the frame velocity. The same confusion follows for the energy density which must strictly obey the Galilean principle. Note, that we may not consider (3) as a Hamiltonian, because the Hamilton function in C-system coincides with the dispersion law in accordance with the Hamilton equation $\dot{\mathbf{r}'} = \mathbf{v} = \frac{\partial\varepsilon}{\partial\mathbf{k}}$.

These shortcomings have been removed in our previous works [1,2,7] (see also [5,6]) where transformations to replace the Galilean ones have been deduced and the quasiparticle mechanics equations in Hamiltonian form have been presented. This enabled us to derive a Boltzmann-like kinetic equation valid in the whole Brillouin zone. We have deduced a self-consistent set of equations for electrons in metals, taking into account some special features of the problem as the quasineutrality condition and the neglection of the displacement current in Maxwell's eqs. The magnetic permeability μ has also been taken constant ($\mu = 1$).

These approximations were good enough to develop the electron plasma hydrodynamics in crystalline metals as well as to consider magnetohydrodynamic effects [8].

The problem is more complicated in bad conductors and semiconductors, as well as at higher frequences, when the displacement current cannot be neglected, the quasineutrality condition does not hold and ϵ and μ are functions of the deformation. We give in this work the solution of this general case. The only assumption is that the constitute relations are linear, i.e. $D_i = \hat{\epsilon}_{ik} E_k$, $B_i = \hat{\mu}_{ik} H_k$.

In the present work we shall consider electrons having in mind that the theory is valid for arbitrary quasiparticles. We suppose for simplicity that the crystal considered is isotropic in its undeformed state. This means that ϵ and μ are taken scalars, but their derivatives with respect to coordinates are matrices and depend on the lattice symmetry. It is easy to generalize all the results for the case where ϵ and μ are tensors of second rank. No essentially new results should be obtained for this case, but the corresponding relations are more cumbersome.

Finally, we would like to note that the problem considered is related to some old questions about the electromagnetic forces acting to the body, some specific features of electrodynamics in moving media, the form of electromagnetic stress tensor in condensed matter, the role of momentum and quasimomentum etc. Since we derive the full set of dynamics equations all the forces are taken into account in a selfconsistent way.

The paper is organized in the following way. In Sec. 2. we introduce new variables (discrete coordinates and ivariant quasimomentum), write evolution equations for lattice vectors in the real and reciprocal spaces and express in new notation metrical tensors, deformation tensor etc. In Sec. 3. we reproduce briefly some of our previous results on the Hamilton eqs. and kinetic equation we need for this work. In Sec. 4 we deduce the full set of equations which describes the behaviour of quasiparticles in deformable solids in electromagnetic fields.

II. NOTATION

Following our previous works [1,2] (see also [5–7]) we consider a lattice with primitive vectors $\mathbf{a}_\alpha,\ \alpha = 1, 2, 3$ and introduce discrete coordinates N^α as the number of steps (each being equal to $\mathbf{a}_\alpha$) in the lattice from the origin to a given point. In this notation the differential coordinates $d\mathbf{r}$ which are considered as physically infinitesimal (i.e. large compared to the lattice constants but small compared to any macroscopic distance of interest) can be written in the form:

$$d\mathbf{r} = \mathbf{a}_\alpha dN^\alpha + \dot{\mathbf{u}}dt$$

or

$$dN^\alpha = \mathbf{a}^\alpha d\mathbf{r} - \mathbf{a}^\alpha \dot{\mathbf{u}}dt$$

where $\mathbf{a}^\alpha$ are the reciprocal lattice vectors which satisfy the relations

$$\mathbf{a}_\alpha \mathbf{a}^\beta = \delta_\alpha^\beta, \qquad a_{\alpha i} a_k^\alpha = \delta_{ik}. \tag{7}$$

It is seen from the above equations that

$$\mathbf{a}^\alpha = \nabla N^\alpha, \qquad \mathbf{a}_\alpha = \frac{\partial \mathbf{r}}{\partial N^\alpha}, \qquad \dot{\mathbf{u}} = -\mathbf{a}_\alpha \dot{N}^\alpha. \tag{8}$$

The time-evolution equations for the vectors $\mathbf{a}_\alpha$ and $\mathbf{a}^\alpha$ can be deduced from plain geometrical considerations (Appendix 1) and written as follows [1]:

$$\dot{\mathbf{a}}_\alpha + (\dot{\mathbf{u}}\nabla)\mathbf{a}_\alpha - (\mathbf{a}_\alpha \nabla)\dot{\mathbf{u}} = 0 \tag{9}$$

$$\dot{\mathbf{a}}^\alpha + \nabla(\mathbf{a}^\alpha \dot{\mathbf{u}}) = 0. \tag{10}$$

In the notation used the metrical tensors in the real and reciprocal space are, respectively:

$$g_{\alpha\beta} = \mathbf{a}_\alpha \mathbf{a}_\beta, \qquad g^{\alpha\beta} = \mathbf{a}^\alpha \mathbf{a}^\beta. \tag{11}$$

Then the lattice cell volume equals $g^{1/2}$, where $g = \det g_{\alpha\beta}$.

The relations between the components of the metrical tensors and the deformation tensor u_{ik} follow from the expression for the interval:

$$ds^2 = g_{\alpha\beta} dN^\alpha dN^\beta. \tag{12}$$

The squared interval between the same points in the undeformed crystal is

$$ds_0^2 = \overset{\circ}{g}_{\alpha\beta}\, dN^\alpha dN^\beta.$$

Hence

$$ds^2 - ds_0^2 = 2w_{\alpha\beta} dN^\alpha dN^\beta \tag{13}$$

where

$$w_{\alpha\beta} = \frac{1}{2}(g_{\alpha\beta} - \overset{\circ}{g}_{\alpha\beta}) \tag{14}$$

plays the role of the deformation tensor in our notation. The invariance of the interval yields:

$$w_{\alpha\beta} dN^\alpha dN^\beta = w_{ik} dx^i dx^k. \tag{15}$$

Taking into account relations (7) and (8) one obtains

$$w_{ik} = w_{\alpha\beta} a_i^\alpha a_k^\beta. \tag{16}$$

In order to obtain the components u_{ik} of the tensor of small deformations (2) as well as to find the relation between the deformation vector $\mathbf{u} = \mathbf{r} - \mathbf{r}_0$ and the discrete coordinates N^α introduced, let us note that the quantities $\partial u_i / \partial x_k$ obviously coincide with the matrix elements α_{ik} which describe the coordinate transformations

$$x_i = \overset{\circ}{x}_i + \alpha_{ik}\, \overset{\circ}{x}_k \tag{17}$$

and, consequently, the lattice vector transformations:

$$a_{\alpha i} = \overset{\circ}{a}_{\alpha i} + \alpha_{ik}\ \overset{\circ}{a}_{\alpha k}\ . \tag{18}$$

If the deformations are small, then,

$$\mathbf{a}_\alpha = \overset{\circ}{\mathbf{a}}_\alpha + \delta\mathbf{a}_\alpha, \qquad \mathbf{a}^\beta = \overset{\circ}{\mathbf{a}}{}^\beta + \delta\mathbf{a}^\beta. \tag{19}$$

Multiplying these two equations and taking into account relations (7) and (17) yield in a linear approximation with respect to $\delta \mathbf{a}^\beta$

$$\delta a_{\alpha i} = -(\overset{\circ}{a}_{\beta i}\, \delta a_k^\beta) \quad \overset{\circ}{a}_{\alpha k} = \alpha_{ik}\, \overset{\circ}{a}_{\alpha k}\ . \tag{20}$$

On the other hand the change of the discrete coordinates owing to the deformation can be written in the form:

$$N^\alpha = N_0^\alpha - w^\alpha. \tag{21}$$

where w^α is the deviation from the value N_0^α in the ideal undeformed lattice. Taking into account (8) one obtains

$$\mathbf{a}^\alpha = \nabla N^\alpha = \overset{\circ}{\mathbf{a}}{}^\alpha - \nabla w^\alpha \tag{22}$$

and hence,

$$\delta a_i^\alpha = -\frac{\partial w^\alpha}{\partial x_i}. \tag{23}$$

It follows from (20) and (23) that

$$\frac{\partial u_i}{\partial x_k} = \alpha_{ik} = \overset{\circ}{a}_{\alpha i}\, \frac{\partial w^\alpha}{\partial x_k} = \frac{\partial\, \overset{\circ}{a}_{\alpha i}\, w^\alpha}{\partial x_k} \tag{24}$$

and therefore the deformation vector $\mathbf{u}$ and the deformation tensor u_{ik} in this linear case are related to their discrete coordinate analogies, w^α and $\overset{\circ}{w}_{\alpha\beta} = \frac{1}{2}\left(\frac{\partial w_\alpha}{\partial N^\beta} + \frac{\partial w_\beta}{\partial N^\alpha}\right)$, in the following way:

$$\mathbf{u} = \overset{\circ}{\mathbf{a}}_\alpha\, w^\alpha, \qquad u_{ik} = \overset{\circ}{w}_{\alpha\beta}\, \overset{\circ}{a}_i^\alpha\ \overset{\circ}{a}_k^\beta \tag{25}$$

where $w_\alpha = \overset{\circ}{g}_{\alpha\beta}\, w^\beta$ are the covariant components of $\mathbf{w}$.

In the same way one can obtain the full deformation tensor components [5]:

$$w_{\alpha\beta} = \frac{1}{2}\left(\overset{\circ}{\mathbf{a}}_\alpha\, \frac{\partial \mathbf{u}}{\partial N^\beta} + \overset{\circ}{\mathbf{a}}_\beta\, \frac{\partial \mathbf{u}}{\partial N^\alpha} + \frac{\partial \mathbf{u}}{\partial N^\alpha}\frac{\partial \mathbf{u}}{\partial N^\beta}\right) \tag{26}$$

III. DYNAMICS AND KINETICS OF QUASIPARTICLES

The starting point when deriving Hamilton equations for quasiparticles is that in the co-moving frame (C-system) the dispersion law $\varepsilon(\mathbf{k}, g^{\alpha\beta})$ coincides both with Hamiltonian and with energy. Hence, the equations of motion in C-system can be written in the form

$$\dot{\mathbf{r}}' = \frac{\partial \varepsilon}{\partial \mathbf{k}}, \qquad \dot{\mathbf{k}} = -\frac{\partial \varepsilon}{\partial \mathbf{r}'}. \tag{27}$$

We have to determine Hamiltonian $H(\mathbf{p}, \mathbf{r}, t)$ as a function of the coordinates $\mathbf{r}$ and quasimomentum $\mathbf{p}$ in L-system in a way to have canonical equations:

$$\dot{\mathbf{r}} = \frac{\partial H}{\partial \mathbf{p}}, \qquad \dot{\mathbf{p}} = -\frac{\partial H}{\partial \mathbf{r}}. \tag{28}$$

According to the general theory of Hamilton mechanics, the Hamiltonian and the momentum can be obtained as derivatives of the action $S(\mathbf{r}, t)$ with respect to the time t and the coordinates $\mathbf{r}$. Equations (27) show that the variables $\mathbf{k}$ and $\mathbf{r}'$ are canonically conjugate. However, if one considers $\mathbf{r}'$ as a continuos variable, then the corresponding quantity $\mathbf{k}$ has to be considered as momentum. Quasimomentum has to be conjugate to some discrete coordinate. Such coordinates, $\mathbf{N}$ (with components N^α), were introduced in Sec.2. Hence, we are able to take the action $S(\mathbf{N}, t)$ as a function of these coordinates and the time. Then we can determine a Hamiltonian $H(\boldsymbol{\kappa}, \mathbf{N}, t)$ and a new quasimomentum $\boldsymbol{\kappa}$ as follows:

$$H(\boldsymbol{\kappa}, \mathbf{N}, t) = -\left(\frac{\partial S}{\partial t}\right)_{\mathbf{N}} \qquad \kappa_\alpha = \left(\frac{\partial S}{\partial N^\alpha}\right)_t. \tag{29}$$

We call $\boldsymbol{\kappa}$ *the invariant quasimomentum*, because all physical quantities, written as functions of $\boldsymbol{\kappa}$, are periodic with a constant period 2π (not $2\pi\mathbf{a}^\alpha$).

Let us consider the local dispersion law $\varepsilon(\boldsymbol{\kappa}, g^{\alpha\beta})$ as a function of the invariant quasimomentum and the metrical tensor $g^{\alpha\beta}$. Since it coincides with the Hamiltonian, one has

$$\varepsilon(\boldsymbol{\kappa}, g^{\alpha\beta}, t) = -\left(\frac{\partial S}{\partial t}\right)_{\mathbf{N}} \qquad \kappa_\alpha = \left(\frac{\partial S}{\partial N^\alpha}\right)_t. \tag{30}$$

Now let us consider a real electron which is executing a quasiclassical motion. Its wave function in the new variables has the form

$$\psi(\mathbf{N},t) \sim exp\left\{\frac{i}{\hbar}S_0(\mathbf{N},t)\right\} \tag{31}$$

where $S_0(\mathbf{N},t)$ is the classical action. The transformation law for the action follows from the transformation properties of the phase of the wave function under Galilean transformations [9]:

$$S = S_0 + m\dot{\mathbf{u}}\mathbf{r} - m\dot{u}^2/2 \tag{32}$$

where m is the mass of a free particle.

The Hamiltonian and the quasimomentum in the Laboratory frame can now be obtained as follows [2]:

$$\mathbf{p} = \left(\frac{\partial S}{\partial \mathbf{r}}\right)_t = \mathbf{a}^\alpha k_\alpha + m\dot{\mathbf{u}} \tag{33}$$

$$H(\mathbf{p},\mathbf{r},t) = -\left(\frac{\partial S}{\partial t}\right)_{\mathbf{r}} = \varepsilon + \mathbf{p}\dot{\mathbf{u}} - \frac{m\dot{u}^2}{2} \tag{34}$$

where $\varepsilon = \varepsilon(\mathbf{a}_\alpha(\mathbf{p} - m\dot{\mathbf{u}}), g^{\alpha\beta})$ is a periodic function of $\mathbf{p}$ with periods $2\pi\hbar\mathbf{a}^\alpha$ determined by the reciprocal lattice vectors corresponding to the deformed local lattice. This is the reason to call $\mathbf{p}$ *the quasimomentum of the quasiparticle in L-system.*

It follows from (33) that

$$k_\alpha = \mathbf{a}_\alpha(\mathbf{p} - m\dot{\mathbf{u}}) = \mathbf{k}\mathbf{a}_\alpha. \tag{35}$$

Hence, the invariant quasimomentum components are equal to the scalar product of the usual quasimomentum $\mathbf{k}$ in C-system and the primitive vectors of the locally deformed lattice.

The energy $\mathcal{E}$ of a quasiparticle in L-system obeys the Galilean law:

$$\mathcal{E} = \frac{m\dot{u}^2}{2} + m\dot{\mathbf{u}}\frac{\partial\varepsilon}{\partial\mathbf{p}} + \varepsilon = \frac{m\dot{u}^2}{2} + \mathbf{p}_0\dot{\mathbf{u}} + \varepsilon \tag{36}$$

where $\mathbf{p}_0 = m\dfrac{\partial\varepsilon}{\partial\mathbf{p}}$ is the average momentum (the mass flow) in C-system.

Equations (33)-(36) replace the Galilean transformations, which are not valid for quasiparticles because of the priviged of the crystal lattice frame.

We are able now to write the Boltzmann equation:

$$\frac{\partial f}{\partial t}+\frac{\partial f}{\partial \mathbf{r}}\frac{\partial H}{\partial \mathbf{p}}-\frac{\partial f}{\partial \mathbf{p}}\left(\frac{\partial H}{\partial \mathbf{r}}-\mathbf{F}\right)=\hat{I}f \tag{37}$$

where $\hat{I}$ is the collision operator. Note, that this equation becomes well defined for quasiparticles only after obtaining the Hamiltonian and the Hamilton equations.

It has been shown in [2] that this equation is compatible with the requirement of periodicity. This can be seen also from its form if $f(\boldsymbol{\kappa},\mathbf{r},t)$ is taken as a function of the invariant quasimomentum $\boldsymbol{\kappa}$ and the quantities $\mathbf{p}$ and H are substituted from (33) and (34). This yields:

$$\frac{\mathrm{d}f}{\mathrm{d}t}+\mathbf{a}_\alpha\frac{\partial\varepsilon}{\partial\kappa_\alpha}\left(\frac{\partial f}{\partial\mathbf{r}}\right)_{\boldsymbol{\kappa}}-\mathbf{a}_\alpha\frac{\partial f}{\partial\kappa_\alpha}\left\{m\frac{\mathrm{d}\dot{\mathbf{u}}}{\mathrm{d}t}+\left(\frac{\partial\varepsilon}{\partial\mathbf{r}}\right)_{\boldsymbol{\kappa}}-m\frac{\partial\varepsilon}{\partial\kappa_\beta}\mathbf{a}_\beta\times\operatorname{curl}\dot{\mathbf{u}}-\mathbf{F}\right\}=\hat{I}f \tag{38}$$

where $\dfrac{\mathrm{d}}{\mathrm{d}t}=\dfrac{\partial}{\partial t}+(\dot{\mathbf{u}}\nabla)$ and all quantities are differentiated with respect to the coordinates and the time at constant $\boldsymbol{\kappa}$.

The term $m\dfrac{\mathrm{d}\dot{\mathbf{u}}}{\mathrm{d}t}$ takes into account noninertial properties of the local frame. This is the term which is responsible for the Stewart-Tolman effect in metals.

The term

$$m\frac{\partial\varepsilon}{\partial\kappa_\beta}\mathbf{a}_\beta\times\operatorname{curl}\dot{\mathbf{u}} \tag{39}$$

is of *essentially new kind* and cannot be obtained in linear theories. It is proportional to the bare mass m and, hence, is also responsible for noninertial effects.

The Lorentz force in our notation has the form

$$\mathbf{F}=-e\mathbf{E}-\frac{e}{c}\frac{\partial\varepsilon}{\partial\kappa_\alpha}\mathbf{a}_\alpha\times\mathbf{B}-\frac{e}{c}\dot{\mathbf{u}}\times\mathbf{B} \tag{40}$$

where $\mathbf{E}$ and $\mathbf{B}$ are the strengths of the electric field and magnetic induction, respectively. Substituting (40) into (38) yields the kinetic equation for charged quasiparticles with a charge $-e$ ($e>0$):

$$\frac{\mathrm{d}f}{\mathrm{d}t} + \mathbf{a}_\alpha \left(\frac{\partial f}{\partial \mathbf{r}}\right)_{\boldsymbol{\kappa}} \frac{\partial \varepsilon}{\partial \kappa_\alpha} - \mathbf{a}_\alpha \frac{\partial f}{\partial \kappa_\alpha} \left\{ \left(\frac{\partial \varepsilon}{\partial \mathbf{r}}\right)_{\boldsymbol{\kappa}} + e\tilde{\mathbf{E}} + \frac{e}{c} \frac{\partial \varepsilon}{\partial \kappa_\beta} \mathbf{a}_\beta \times \tilde{\mathbf{B}} \right\} = \hat{I} f, \tag{41}$$

where

$$e\tilde{\mathbf{E}} = e\mathbf{E} + \frac{e}{c}\dot{\mathbf{u}} \times \mathbf{B} + m\frac{\mathrm{d}\dot{\mathbf{u}}}{\mathrm{d}t}, \qquad \tilde{\mathbf{B}} = \mathbf{B} - \frac{mc}{e} \operatorname{curl} \dot{\mathbf{u}}. \tag{42}$$

The first expression in (42) consists of two parts — the electrical force $e\mathbf{E}'$ where $\mathbf{E}' = \mathbf{E} + \frac{1}{c}\dot{\mathbf{u}} \times \mathbf{B}$ is the field in the co-moving frame, and the inertial force $m\frac{\mathrm{d}\dot{\mathbf{u}}}{\mathrm{d}t}$. The second relation in (42) can also be written in the form $\tilde{\mathbf{B}} = \operatorname{curl}(\mathbf{A} - \frac{c}{e} m\dot{\mathbf{u}})$ where $\mathbf{A}$ is the vector potential in an agreement with the well-known rule of replacement of the particle momentum in a magnetic field $\mathcal{P} \to \mathcal{P} - \frac{e}{c}\mathbf{A}$ where now $\mathcal{P} = m\dot{\mathbf{u}}$.

We shall use also another form of the kinetic equation. The reason is that further on we need to integrate some physical quantities over the Brillouin zone, take integrals by parts as well as differentiate with respect to the coordinates and the time. However, the Brillouin zone boundaries are functions not only of the deformation at given instant, but also on the local lattice velocity. The integration over the Brillouin zone does not commutate with the differentiation with respect to the time and the coordinates. As a result some fluxes appear through the zone boundaries. This effect is important for nonequilibrium systems, open Fermi-surfaces as well as for other cases when the partition function or its derivatives do not vanish on the zone boundaries. This inconvenience can be passed over by introducing a renormalized partition function

$$\varphi(\boldsymbol{\kappa}, \mathbf{r}, t) = f/\sqrt{g}. \tag{43}$$

The kinetic equation for $\varphi(\boldsymbol{\kappa}, \mathbf{r}, t)$ has the form

$$\dot{\varphi} + \operatorname{div} \left\{ \left(\dot{\mathbf{u}} + \mathbf{a}_\alpha \frac{\partial \varepsilon}{\partial \kappa_\alpha} \right) \varphi \right\} - \mathbf{a}_\alpha \frac{\partial}{\partial \kappa_\alpha} \left\{ \varphi \left(\nabla \varepsilon + m \frac{\mathrm{d}\dot{\mathbf{u}}}{\mathrm{d}t} \right) - m\varphi \frac{\partial \varepsilon}{\partial \kappa_\beta} \mathbf{a}_\beta \times \operatorname{curl} \dot{\mathbf{u}} - \mathbf{F}\varphi \right\} = \hat{I}\varphi. \tag{44}$$

In this notation the differentiation with respect to t and $\mathbf{r}$ is carried out at constant $\boldsymbol{\kappa}$ and hence commutates with $\int d^3\kappa$. Results obtained by such a procedure can easily be

rewritten in the previously adopted variables by the following substitutions

$$\mathbf{a}_\alpha \frac{\partial}{\partial \kappa_\alpha} \leftrightarrow \frac{\partial}{\partial \mathbf{k}} \leftrightarrow \frac{\partial}{\partial \mathbf{p}}$$

$$\begin{aligned} \langle f \ldots \rangle \equiv \int \frac{d^3p}{(2\pi\hbar)^3} f(\mathbf{p},\mathbf{r},t)\ldots &= \frac{1}{\sqrt{g}} \int \frac{d^3\kappa}{(2\pi\hbar)^3} f(\mathbf{p},\mathbf{r},t)\ldots \\ = \int \frac{d^3\kappa}{(2\pi\hbar)^3} \varphi(\boldsymbol{\kappa},\mathbf{r},t)\ldots &\equiv \langle\langle \varphi \ldots \rangle\rangle \,, \end{aligned} \tag{45}$$

$$\int d^3r \ldots = \int d^3N^\alpha g^{1/2} \ldots \tag{46}$$

IV. CONSERVATION LAWS AND DYNAMICS EQUATIONS

In order to avoid cumbersome expressions we shall make our consideration in three steps. First we write the conservation laws and consider the problem in a general form without taking into account the explicit form of the Maxwell's equations and electromagnetic forces. Then we consider Maxwell's equations and constitute relations for moving media and finally we combine the results and obtain the full selfconsistent set of equations.

We start with the conservation laws.

The continuity equation for quasiparticles is

$$m\dot{n} + \operatorname{div} \mathbf{j}_0 = 0 \tag{47}$$

where

$$n = \langle\langle \varphi \rangle\rangle = \langle f \rangle, \quad \mathbf{j}_0 = m \left\langle \frac{\partial H}{\partial \mathbf{p}} f \right\rangle = m \left\langle \frac{\partial \varepsilon}{\partial \mathbf{p}} f \right\rangle + mn\dot{\mathbf{u}}. \tag{48}$$

This equation follows directly from the kinetic equation [1].

The total mass current is

$$\mathbf{J}_0 = \rho\dot{\mathbf{u}} + \mathbf{j}_0', \tag{49}$$

where

$$\mathbf{j}_0' = m\left\langle \frac{\partial \varepsilon}{\partial \mathbf{p}} f \right\rangle \tag{50}$$

is the quasiparticle mass current with respect to the lattice and $\rho = \rho_0 + mn$ is the full mass density written as a sum of the lattice mass density ρ_0 and the quasiparticle mass density.

The quantities ρ and $\mathbf{J}_0$ satisfy the mass continuity equation

$$\dot{\rho} + \operatorname{div} \mathbf{J}_0 = 0. \tag{51}$$

The full momentum $\mathbf{J}$ is a sum of $\mathbf{J}_0$ and the field momentum $\mathbf{g}$:

$$\mathbf{J} = \mathbf{J}_0 + \mathbf{g}. \tag{52}$$

Note, that in this case *the full momentum does not coincide with the mass flow!*

Our aim is to determine momentum and energy fluxes Π_{ik} and $\mathbf{Q}$ in such a way as to satisfy the continuity equation (51), the momentum conservation law

$$\dot{J}_i + \nabla_k \Pi_{ik} = 0, \tag{53}$$

and the energy conservation law

$$\dot{E} + \operatorname{div} \mathbf{Q} = 0. \tag{54}$$

The energy in L-system is given by the expression

$$E = -\frac{1}{2}\rho_0 \dot{\mathbf{u}}^2 + E_0(g^{\alpha\beta}) + \langle\langle \mathcal{E}\varphi \rangle\rangle + W, \tag{55}$$

where $E_0(g^{\alpha\beta})$ is the strain energy in C-system, and W is the field energy.

The time derivative of the energy (55) is then

$$\begin{aligned} \dot{E} = {} & \rho \dot{\mathbf{u}}\ddot{\mathbf{u}} + \frac{1}{2}\dot{\rho}\dot{u}^2 + \frac{\partial}{\partial t}\langle\langle \varepsilon\varphi \rangle\rangle + m\ddot{\mathbf{u}}\mathbf{a}_\alpha \left\langle\left\langle \varphi \frac{\partial \varepsilon}{\partial k_\alpha} \right\rangle\right\rangle \\ & + m\dot{\mathbf{u}}\dot{\mathbf{a}}_\alpha \left\langle\left\langle \varphi \frac{\partial \varepsilon}{\partial k_\alpha} \right\rangle\right\rangle + m\dot{\mathbf{u}}\mathbf{a}_\alpha \left\langle\left\langle \varphi \frac{\partial \dot{\varepsilon}}{\partial k_\alpha} \right\rangle\right\rangle + m\dot{\mathbf{u}}\mathbf{a}_\alpha \left\langle\left\langle \dot{\varphi} \frac{\partial \varepsilon}{\partial k_\alpha} \right\rangle\right\rangle + \dot{W}. \end{aligned} \tag{56}$$

The time derivative of the elastic energy $E_0(g^{\alpha\beta})$ can be taken in the following way.

$$\dot{E}_0 = \frac{\partial E_0}{\partial g^{\alpha\beta}} \dot{g}^{\alpha\beta} = -\sigma_{\alpha\beta} \dot{a}_i^\alpha a_i^\beta \tag{57}$$

where

$$\sigma_{\alpha\beta} = -2\frac{\partial E_0}{\partial g^{\alpha\beta}}. \tag{58}$$

Substituting $\dot{a}_i^\alpha$ from the evolution equation (10) in (57) yields

$$\dot{E}_0 = \sigma_{\alpha\beta} a_i^\beta \left(\frac{\partial a_i^\alpha}{\partial x_k} \dot{u}_k + a_k^\alpha \frac{\partial \dot{u}_k}{\partial x_i} \right). \tag{59}$$

On the other hand

$$\nabla_k E_0 = \frac{\partial E_0}{\partial g^{\alpha\beta}} \nabla g^{\alpha\beta} = -\sigma_{\alpha\beta} a_i^\beta \frac{\partial a_i^\alpha}{\partial x_k}. \tag{60}$$

It follows from (59) and (60) that

$$\dot{E}_0 = \sigma_{\alpha\beta} a_i^\alpha a_k^\beta \frac{\partial \dot{u}_i}{\partial x_k} - \dot{\mathbf{u}} \nabla E_0. \tag{61}$$

The time derivative of the full momentum (52) gives

$$0 = -\dot{\mathbf{J}} + \dot{\rho}\dot{\mathbf{u}} + \rho\ddot{\mathbf{u}} + m\dot{\mathbf{a}}_\alpha \left\langle \left\langle \frac{\partial \varepsilon}{\partial k_\alpha} \varphi \right\rangle \right\rangle + m\mathbf{a}_\alpha \left\langle \frac{\partial \varepsilon}{\partial k_\alpha} \dot{\varphi} \right\rangle + m\mathbf{a}_\alpha \left\langle \left\langle \frac{\partial \dot{\varepsilon}}{\partial k_\alpha} \varphi \right\rangle \right\rangle + \dot{\mathbf{g}}. \tag{62}$$

Multiplying (62) by $-\dot{\mathbf{u}}$ and adding the result to the right hand side of (56) yield

$$\dot{E} = \dot{\mathbf{u}}\dot{\mathbf{J}} - \frac{1}{2}\dot{\rho}\dot{\mathbf{u}}^2 + \sigma_{\alpha\beta} a_i^\alpha a_k^\beta \frac{\partial \dot{u}_i}{\partial x_k} - m\ddot{\mathbf{u}}\mathbf{a}_\alpha \left\langle \left\langle \varphi \frac{\partial \varepsilon}{\partial k_\alpha} \right\rangle \right\rangle - \dot{\mathbf{u}} \nabla E_0 + \dot{W} - \dot{\mathbf{u}}\dot{\mathbf{g}} + \frac{\partial}{\partial t} \langle\langle \varepsilon\varphi \rangle\rangle. \tag{63}$$

The last term is considered in Appendix 2. Substituting the time derivatives $\dot{\mathbf{J}}$ and $\dot{\rho}$ by means of (53) and (51) one obtains after cumbersome calculations:

$$\begin{aligned}
&\dot{E} + \nabla_k \left\{ \frac{1}{2}\rho\dot{u}^2 \dot{u}_k + \dot{u}_i \left(\Pi_{ik} - \rho\dot{u}_i\dot{u}_k + E_0\delta_{ik} + \langle\langle \varepsilon f \rangle\rangle \delta_{ik} \right) \right. \\
&\quad \left. - \frac{1}{2} m\dot{u}^2 \left\langle \frac{\partial \varepsilon}{\partial p_k} f \right\rangle + \left\langle \varepsilon \frac{\partial \epsilon}{\partial p_k} f \right\rangle \right\} \\
&\quad = \frac{\partial \dot{u}_i}{\partial x_k} \left\{ \Pi_{ik} - \rho\dot{u}_i\dot{u}_k + \sigma_{ik} - \langle \lambda_{ik} f \rangle + E_0\delta_{ik} \right. \\
&\quad \left. - \dot{u}_i j_{0i} - \dot{u}_k j_{0i} \right\} + \langle \varepsilon \hat{I} f \rangle + \left\langle \mathbf{F} \frac{\partial \varepsilon}{\partial \mathbf{p}} f \right\rangle + \dot{W} - \dot{\mathbf{u}}\dot{\mathbf{g}}
\end{aligned} \tag{64}$$

where

$$\lambda_{ik} = 2\frac{\partial \varepsilon}{\partial g^{\alpha\beta}} a_i^\alpha a_k^\beta \qquad \sigma_{ik} = -2\frac{\partial E_0}{\partial g^{\alpha\beta}} a_i^\alpha a_k^\beta. \tag{65}$$

The last three terms in (64) describe the change of the field energy, field momentum and the effect of external forces. They depend on the concrete type of interaction.

If there are no external fields then the last three terms in (64) should be omitted and the energy and momentum fluxes are:

$$Q_i = E\dot{u}_i + \left\langle \varepsilon \frac{\partial H}{\partial p_i} f \right\rangle - \frac{1}{2}\dot{u}^2 J_i + \Pi_{ik}\dot{u}_k \tag{66}$$

$$\Pi_{ik} = -(\sigma_{ik} + E_0\delta_{ik}) + \rho\dot{u}_i\dot{u}_k + \langle \lambda_{ik} f \rangle - m\left\langle f \frac{\partial \varepsilon}{\partial p_i}\frac{\partial \varepsilon}{\partial p_k} \right\rangle + m\left\langle f \frac{\partial H}{\partial p_i}\frac{\partial H}{\partial p_k} \right\rangle. \tag{67}$$

The momentum flux tensor consists of two parts, L_{ik} and P_{ik}, which correspond to the contributions of the lattice and quasiparticles respectively:

$$L_{ik} = -(\sigma_{ik} + E_0\delta_{ik}) + \rho_0\dot{u}_i\dot{u}_k \tag{68}$$

$$P_{ik} = \langle \lambda^0_{ik} f \rangle + m\left\langle f \frac{\partial H}{\partial p_i}\frac{\partial H}{\partial p_k} \right\rangle \tag{69}$$

where

$$\langle \lambda^0_{ik} f \rangle = \langle \lambda_{ik} f \rangle - m\left\langle f \frac{\partial \varepsilon}{\partial p_i}\frac{\partial \varepsilon}{\partial p_k} \right\rangle$$

is the quasiparticle momentum flux tensor in the system of centre of mass while $\langle \lambda_{ik} f \rangle$ corresponds to the co-moving frame. It can be shown (Appendix 3) that the sume $\sigma_{ik} + E_0\delta_{ik}$ corresponds (but coinsides in linear approximation only) to the stress tensor of the linear elasticity theory and turns into pressure for isotropic media.

Finally, the equation of the elasticity theory for an elastic crystalline body with quasiparticle excitations in the absence of external fields takes the form:

$$\frac{\partial}{\partial t}(\rho\dot{u}_i) = -\frac{\partial \Pi_{ik}}{\partial x_k} - \frac{\partial j_{0i}}{\partial t}. \tag{70}$$

The last term in the right-hand side describes the force which appears when varying the quasiparticle mass current with respect to the lattice.

Let us now consider the effect of the electromagnetic field. The Maxwell's equations are:

$$\operatorname{curl}\mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}\,, \qquad \operatorname{curl}\mathbf{H} = \frac{4\pi}{c}\mathbf{j}_e + \frac{1}{c}\frac{\partial \mathbf{D}}{\partial t}\,, \tag{71}$$

$$\operatorname{div}\mathbf{D} = 4\pi q, \qquad \operatorname{div}\mathbf{B} = 0, \tag{72}$$

where $\mathbf{j}_e = \mathbf{j}'_e + q\dot{\mathbf{u}}$, $q = q_0 - en$ is the charge density (q_0 being the lattice charge) and

$$\mathbf{j}'_e = -\frac{e}{m}\mathbf{j}'_0, \quad e > 0 \tag{73}$$

is the electron current density in the co-moving frame.

We have now to take into account the field terms in (64) containing the densities of the field energy W, the field momentum $\mathbf{g}$ and the Lorentz force $\mathbf{F}$.

Before going on we would like to mention that the extraneous charges q_0 have to be considered as a separate system. They are not accounted in the Boltzmann equation as well as in the Lorentz force (40). Therefore, one needs some additional equations. In metals, this additional equation is the quasineutrality condition [2] $q_0 = en$. This is a good approximation also for semiconductors at low frequencies. The behaviour of the systems of electrons and other charges in the crystal depends on the problem considered. This is not the aim of our work. The presence of extraneous charges makes the whole system open and conservation laws (47)-(55) do not present a full system. One has to take into account both mechanical work and that of the induced forces. If the only current carriers are electrons, then one has to put $q = 0$. However, in order to keep the general form of Maxwell's equations we shall compensate the missing contribution [1] to the time derivative of the field energy by a term $\dot{w} = -\dot{\mathbf{u}}\left(q\mathbf{E} + \frac{1}{c}\mathbf{j_e} \times \mathbf{B}\right)$.

Therefore, the full field contribution is:

$$\dot{W}_f = \left\langle \mathbf{F}\frac{\partial \varepsilon}{\partial \mathbf{p}} f \right\rangle + \dot{W} - \dot{\mathbf{u}}\dot{\mathbf{g}} + \dot{w}. \tag{74}$$

Maxwell's eqs. (71) and (72) are written in L-system. They have to be supplemented

[1] Actually, if one is only interested in stress-tensor components then charges and current can be omitted (c.f. Ref. [10], §34).

by constitute relations. However, one has to keep in mind that these relations have their known simple form only in the co-moving frame. In that frame one has:

$$\mathbf{D}' = \epsilon \mathbf{E}', \qquad \mathbf{B}' = \mu \mathbf{H}', \tag{75}$$

where

$$\mathbf{D}' = \mathbf{D} + \frac{1}{c}\mathbf{v} \times \mathbf{H}, \qquad \mathbf{E}' = \mathbf{E} + \frac{1}{c}\mathbf{v} \times \mathbf{B} \tag{76}$$

$$\mathbf{B}' = \mathbf{B} - \frac{1}{c}\mathbf{v} \times \mathbf{E}, \qquad \mathbf{H}' = \mathbf{H} - \frac{1}{c}\mathbf{v} \times \mathbf{D} \tag{77}$$

and $\mathbf{v} = \dot{\mathbf{u}}$ is introduced for convenience.

It is easy to see that

$$\mathbf{vD} = \mathbf{vD}' = \epsilon \mathbf{vE}' = \epsilon \mathbf{vE}, \qquad \mathbf{vB} = \mathbf{vB}' = \mu \mathbf{vH}' = \mu \mathbf{vH}. \tag{78}$$

Relations (75)-(78) are exact althogh (76) and (77) coincide in letter to the field transformations with an accuracy to v/c.[2]

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[2] The exact field transformations are

$$\tilde{\mathbf{E}} = \gamma \mathbf{E} + (1-\gamma)(\mathbf{E}.\mathbf{n})\mathbf{n} + \gamma \frac{\mathbf{v}}{c} \times \mathbf{B}, \qquad \tilde{\mathbf{B}} = \gamma \mathbf{B} + (1-\gamma)(\mathbf{B}.\mathbf{n})\mathbf{n} - \gamma \frac{\mathbf{v}}{c} \times \mathbf{E}$$
$$\tilde{\mathbf{D}} = \gamma \mathbf{D} + (1-\gamma)(\mathbf{D}.\mathbf{n})\mathbf{n} + \gamma \frac{\mathbf{v}}{c} \times \mathbf{H}, \qquad \tilde{\mathbf{H}} = \gamma \mathbf{H} + (1-\gamma)(\mathbf{H}.\mathbf{n})\mathbf{n} - \gamma \frac{\mathbf{v}}{c} \times \mathbf{D}$$

where

$$\mathbf{n} = \frac{\mathbf{v}}{v}, \qquad \gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2}$$

These fields obviously satisfy relations (78). Then from the constitute relations in the co-moving frame $\tilde{\mathbf{D}} = \epsilon \tilde{\mathbf{E}},\ \tilde{\mathbf{B}} = \mu \tilde{\mathbf{H}}$ one obtains immediately relations (75)-(77) in the form:

$$\mathbf{D} + \frac{1}{c}\mathbf{v} \times \mathbf{H} = \epsilon \left\{ \mathbf{E} + \frac{1}{c}\mathbf{v} \times \mathbf{B} \right\}, \qquad \mathbf{B} - \frac{1}{c}\mathbf{v} \times \mathbf{E} = \mu \left\{ \mathbf{H} - \frac{1}{c}\mathbf{v} \times \mathbf{D} \right\}$$

When taking the time derivative of the field energy $\dot{W}$ in (74) one has to keep in mind that the permeabilities μ and ϵ in a nonstationary deformed media are functions of space and time. For example, the variation of the electrical part of the field energy in the lattice frame is:

$$\delta W'_E = \frac{1}{4\pi}\mathbf{E}'\delta\mathbf{D}' = \frac{1}{4\pi}\left\{\mathbf{E'^2}\delta\epsilon + \epsilon\mathbf{E}'\delta\mathbf{E}'\right\} = \frac{\mathbf{E'^2}}{8\pi}\delta\epsilon + \delta\frac{\epsilon\mathbf{E^2}}{8\pi}.$$

Therefore, the full variation of W in time can be written in the form:

$$\begin{aligned}
\dot{W} &= \frac{1}{4\pi}(\mathbf{E}\dot{\mathbf{D}} + \mathbf{H}\dot{\mathbf{B}}) = \\
&= \frac{1}{4\pi}\left(\mathbf{E}' - \frac{\mathbf{v}}{c}\times\mathbf{B}\right)\left(\dot{\mathbf{D}}' - \frac{\mathbf{v}}{c}\times\dot{\mathbf{H}}\right) + \frac{1}{4\pi}\left(\mathbf{H}' + \frac{\mathbf{v}}{c}\times\mathbf{D}\right)\left(\dot{\mathbf{B}}' - \frac{\mathbf{v}}{c}\times\dot{\mathbf{E}}\right) = \\
&= \frac{\partial}{\partial t}\frac{\epsilon E'^2 + \mu H'^2}{8\pi} + \frac{E'^2}{8\pi}\dot{\epsilon} + \frac{H'^2}{8\pi}\dot{\mu} + \mathbf{v}(\dot{\mathbf{g}} + \dot{\mathbf{G}}) + \\
&+ \frac{\partial}{\partial t}\frac{\epsilon(\mathbf{v}\times\mathbf{E}')^2 + \mu(\mathbf{v}\times\mathbf{H}')^2}{8\pi c^2} + \frac{(\mathbf{v}\times\mathbf{E}')^2}{8\pi c^2}\dot{\epsilon} + \frac{(\mathbf{v}\times\mathbf{H}')^2}{8\pi c^2}\dot{\mu} + 0(v^3/c^3)
\end{aligned} \tag{79}$$

where

$$\mathbf{g} = \frac{\mathbf{E}\times\mathbf{H}}{4\pi c}, \qquad \mathbf{G} = \frac{\mathbf{D}\times\mathbf{B}}{4\pi c}. \tag{80}$$

From here on we shall restrict our consideration within an accuracy to v/c (neglecting terms $0(v^2/c^2)$. Then

$$\dot{W} - \mathbf{v}\dot{\mathbf{g}} = \frac{\partial}{\partial t}\frac{\epsilon E'^2 + \mu H'^2}{8\pi} + \frac{E'^2}{8\pi}\dot{\epsilon} + \frac{H'^2}{8\pi}\dot{\mu} + \mathbf{v}\dot{\mathbf{G}}. \tag{81}$$

The quantities ϵ and μ are functions of the metrical tensor $g^{\alpha\beta}$. So their time derivatives $\dot{\epsilon}$ and $\dot{\mu}$ can be treated in the same way as the derivative of E_0 (c.f. (57)-(61)). This yields:

$$\dot{\epsilon} = -\epsilon_{ik}\frac{\partial v_i}{\partial x_k} - \mathbf{v}\nabla\epsilon, \qquad \dot{\mu} = -\mu_{ik}\frac{\partial v_i}{\partial x_k} - \mathbf{v}\nabla\mu, \tag{82}$$

where

$$\epsilon_{ik} = 2\frac{\partial\epsilon}{\partial g^{\alpha\beta}}a_i^\alpha a_k^\beta, \qquad \mu_{ik} = 2\frac{\partial\mu}{\partial g^{\alpha\beta}}a_i^\alpha a_k^\beta. \tag{83}$$

Substituting (82) and (83) in (81) and making use of the Poynting theoreme one obtains

$$\dot{W} - \mathbf{v}\dot{\mathbf{g}} = -\mathrm{div}\,\mathbf{S}' - \mathbf{j}'_e\mathbf{E}' - \left(\frac{\mathbf{E}'^2}{8\pi}\epsilon_{ik} + \frac{\mathbf{H}'^2}{8\pi}\mu_{ik}\right)\frac{\partial v_i}{\partial x_k} - \mathbf{v}\left(\frac{\mathbf{E}'^2}{8\pi}\nabla\epsilon + \frac{\mathbf{H}'^2}{8\pi}\nabla\mu\right) + \mathbf{v}\dot{\mathbf{G}}. \tag{84}$$

Neglecting terms of the order $0(v^2/c^2)$ in (79) means that one can replace $\dot{\mathbf{G}}$ by $\dot{\mathbf{G}}'$. The time derivative $\dot{\mathbf{G}}'$ can be transformed using Maxwell's equations in the co-moving frame. This yields:

$$\mathbf{v}\dot{\mathbf{G}}' = -\mathbf{v}\left(q\mathbf{E}' + \frac{1}{c}\mathbf{j}'_e \times \mathbf{B}'\right) + v_i\nabla_k t'_{ik} + \mathbf{v}\left(\frac{\mathbf{E}'^2}{8\pi}\nabla\epsilon + \frac{\mathbf{H}'^2}{8\pi}\nabla\mu\right) \tag{85}$$

where

$$t'_{ik} = \frac{\epsilon}{4\pi}\left(E'_iE'_k - \frac{E'^2}{2}\delta_{ik}\right) + \frac{\mu}{4\pi}\left(H'_iH'_k - \frac{H'^2}{2}\delta_{ik}\right) \tag{86}$$

is the Maxwell's stress tensor in the co-moving frame. It is easy to show using eqs. (76)-(78) that

$$\mathbf{v}\left(q\mathbf{E}' + \frac{1}{c}\mathbf{j}'_e \times \mathbf{B}'\right) = \mathbf{v}\left(q\mathbf{E} + \frac{1}{c}\mathbf{j}_e \times \mathbf{B}\right) = \mathbf{j_e}\mathbf{E} - \mathbf{j'_e}\mathbf{E}'.$$

The term which contains Lorentz force in (74) can be calculated by means of (40):

$$\begin{aligned}\left\langle \mathbf{F}\frac{\partial\varepsilon}{\partial\mathbf{p}}f\right\rangle &= \mathbf{E}\left\langle -e\frac{\partial\varepsilon}{\partial\mathbf{p}}f\right\rangle - \frac{e}{c}\mathbf{B}\left\langle \frac{\partial\varepsilon}{\partial\mathbf{p}}\times\frac{\partial\varepsilon}{\partial\mathbf{p}}f\right\rangle + \frac{1}{c}\dot{\mathbf{u}}\times\mathbf{B}\left\langle -e\frac{\partial\varepsilon}{\partial\mathbf{p}}f\right\rangle \\ &= \mathbf{j}'_e\left(\mathbf{E} + \frac{\dot{\mathbf{u}}}{c}\times\mathbf{B}\right) = \mathbf{j'_e}\mathbf{E}'.\end{aligned} \tag{87}$$

The same term, but with a negative sign exists also in (84). Therefore, the total work related to the Lorentz force (the mechanical one and that of the electromotive forces) equals zero as it should be. The terms related to the extraneous charges cancel for the same reason. Finally,

$$\dot{W}_f = -\mathrm{div}\,\mathbf{S}' + \nabla_k v_i t'_{ik} - T'_{ik}\frac{\partial v_i}{\partial x_k} \tag{88}$$

where

$$T'_{ik} = \frac{1}{4\pi}\left\{\epsilon E'_iE'_k + \frac{E'^2}{2}(\epsilon_{ik} - \epsilon\delta_{ik}) + \mu H'_iH'_k + \frac{H'^2}{2}(\mu_{ik} - \mu\delta_{ik})\right\} \tag{89}$$

Hence, one has to add the term

$$Q_i^f = S_i' - v_k t_{ik}' = S_i - v_i W \tag{90}$$

to the energy flux density in (66), as well as the term $-T_{ik}'$ to the momentum flux tensor Π_{ik} in (67).

The elasticity theory equation takes then the form:

$$\frac{\partial}{\partial t}(\rho \dot{u}_i) = -\frac{\partial L_{ik}}{\partial x_k} - \frac{\partial P_{ik}}{\partial x_k} + \frac{\partial T_{ik}'}{\partial x_k} + \frac{m}{e}\frac{\partial j_{ei}'}{\partial t} - \frac{\partial g_i}{\partial t}\,. \tag{91}$$

The current and the electromagnetic stress tensor $\hat{T}'$ are written in the lattice frame. Note, that the term with the electrical current represents, in fact, the electron mass-flow (the momentum, associated with the current). That part of the electron mass current, which moves together with the latice, is included in ρ in the left-hand side of the equation. The electromagnetic momentum flux tensor in L-system has the form:

$$T_{ik} = \frac{1}{4\pi}\left(\epsilon E_i E_k + \mu H_i H_k\right) + (\epsilon_{ik} - \epsilon\delta_{ik})\frac{E^2}{8\pi} + (\mu_{ik} - \mu\delta_{ik})\frac{H^2}{8\pi} + v_i G_k + v_k G_i - v_i g_k - v_k g_i \tag{92}$$

Obviously, for small velocities one can replace T_{ik}' by the corresponding tensor T_{ik} in L-system.

If electrons in a good conductor (metal) are considered, then the quasineutrality condition holds and the displacement current as well as the field momenta $\mathbf{g}$ and $\mathbf{G}$ have to be put zero. Atually, the two terms in the right-hand side of the Ampere's law can be considered as expansion with respect to the electric field frequency: $\mathbf{j} + \frac{1}{4\pi}\frac{\partial \mathbf{D}}{\partial t} \approx \sigma\mathbf{E} + \frac{\omega\epsilon}{4\pi}\mathbf{E}$. In metals $\sigma \gg \omega\epsilon/(4\pi)$, and the dissplacement current can be neglected. As a result, one has

$$T_{ik}^{metal} = \frac{\mu}{4\pi} H_i H_k + (\mu_{ik} - \mu\delta_{ik})\frac{H^2}{8\pi} \tag{93}$$

This tensor contains an additional term $\frac{H'^2}{8\pi}\mu_{ik}$ compared to our previous works [2,5,6,8], in which the magnetic permeability has been taken constant. As shown in Appendix 3 in case of a noncrystalline body (e.g. fluid) the quantities μ_{ik} and ϵ_{ik} have to be replaced by $\left(\rho\frac{\partial \mu}{\partial \rho}\right)_T \delta_{ik}$ and $\left(\rho\frac{\partial \epsilon}{\partial \rho}\right)_T \delta_{ik}$, correspondingly (comp. Ref. [10], §56).

V. CONCLUSION

In this work we have considered dynamics and kinetics of charged quasiparticles with arbitrary dispersion relations in deformable crystalline structures. We have chosen the most general case of time-varying deformations when the quasimomentum is not a good quantum number, and energy does not coinside to the Hamiltonian. We have derived a full selfconsistent set of equations which consists of the nonlinear elasticity theory equation (91) the kinetic equation (41) or (44) and Maxwell's equations supplemented by constitute relations. The kinetic equation is valid for the whole Brillouin zone and contains a new term responsible for some inertial effects. The elasticity theory equation is derived from the conservation laws written in the most general form. The only approximation is that the electromagnetic field transformations are taken with an accuracy to $0(v^2/c^2)$. Any higher accuracy for the solid-state theory now would be pointless. In such a way the theory presented is exact in the frame of the quasiparticle approach. It can be used for any material (metal, semiconductor, quantum crystal, low-dimensional structures etc.) with linear relations between electromagnetic fields. It is easy to write the corresponding set of equations for more than one type of quasiparticles (e.g. electrons and holes).

In case of electrons in metals the results obtained fit well to those of our previous works [2,5].

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APPENDIX 1

The evolution equation (9) for the primitive vectors $\mathbf{a}_\alpha$ can be deduced from the following consideration (we shall omit the subscript α for conveninence). The lattice vector $\mathbf{a}(\mathbf{r},\mathbf{t})$ at instant t is defined by the two lattice sites $\mathbf{r_1}(\mathbf{t})$ and $\mathbf{r_2}(\mathbf{t})$: $\mathbf{a}=\mathbf{r_2}-\mathbf{r_1}$. After a time interval δt the lattice vector changes to $\mathbf{a}'(\mathbf{r}',t+\delta t)=\mathbf{r}'_\mathbf{2}-\mathbf{r}'_\mathbf{1}$. The new positions of the lattice sites are obviously $\mathbf{r}'_\mathbf{1}=\mathbf{r_1}+\mathbf{v}(\mathbf{r_1})\delta t$, and $\mathbf{r}'_\mathbf{2}=\mathbf{r_2}+\mathbf{v}(\mathbf{r_2})\delta t$ where the velocity of the lattice site $\mathbf{r}$ in the moment t is denoted by $\mathbf{v}(\mathbf{r})=\dot{\mathbf{u}}(\mathbf{r})$. It follows from the last two equations that

$$\mathbf{a}'-\mathbf{a}=(\mathbf{v}(\mathbf{r_2})-\mathbf{v}(\mathbf{r_1}))\delta\mathbf{t}.$$

Taking into account that $\mathbf{v}(\mathbf{r_2})=\mathbf{v}(\mathbf{r_1}+\mathbf{a})=\mathbf{v}(\mathbf{r_1})+(\mathbf{a}\nabla)\mathbf{v}$, and $\mathbf{a}'=\mathbf{a}(\mathbf{r}+\mathbf{v}\delta t,t+\delta t)=\mathbf{a}+\dot{\mathbf{a}}+(\mathbf{v}\nabla)\mathbf{a}$ one obtains

$$\dot{\mathbf{a}}+(\mathbf{v}\nabla)\mathbf{a}-(\mathbf{a}\nabla)\mathbf{v}=\mathbf{0}. \qquad (A1.1)$$

This equation coinsides with (9) in the Notation. It conserves automatically the lattice vectors lines. In fact, the **a**-vector line conservation condition consists in collinearity of **a** and the left-hand side of (A1.1), [11] i.e. in

$$[\dot{\mathbf{a}}+(\mathbf{v}\nabla)\mathbf{a}-(\mathbf{a}\nabla)\mathbf{v}]\times\mathbf{a}=0. \qquad (A1.2)$$

Hence, eq. (A1.1) describes deformations which do not break or cross crystalline lines with equal α. This means that in a crystal lattice free of dislocations the three functions $N^\alpha(\mathbf{r},t)$ are single-valued, and equation (A1.1) desdribes completely the evolution of the lattice configuration.

The evolution equation for the reciprocal lattice vectors $\mathbf{a}^\alpha$ can be obtained from (A1.1) and the relation

$$\mathbf{a}^\alpha=\frac{\partial N^\alpha}{\partial\mathbf{r}}. \qquad (A1.3)$$

The later follows directly from the expression for the physically infinitisimal differential coordinates at given instant $d\mathbf{r}=\mathbf{a}_\alpha dN^\alpha$ and relations (7). Multiplying (A1.1) by $\mathbf{a}^\alpha_k$ yields

$$a^\alpha_k\dot{a}_{\alpha i}+a^\alpha_k(\dot{\mathbf{u}}\nabla)a_{\alpha i}-a^\alpha_k a_{\alpha s}\nabla_s\dot{u}_i=0 \qquad (A1.4)$$

Taking into account relations (7) one has

$$a_{\alpha i}\left(\dot{a}_k^\alpha + (\dot{\mathbf{u}}\nabla)a_k^\alpha\right) + \nabla_k \dot{u}_i = 0 \qquad (A1.5)$$

Multiplying this equation with a_i^β yields

$$\dot{\mathbf{a}}^\alpha + \nabla(\mathbf{a}^\alpha \dot{\mathbf{u}}) = \mathbf{0} \qquad (A1.6)$$

In obtaining (A1.6) we have taken into account that $\nabla_i a_k^\alpha - \nabla_k a_i^\alpha = 0$ as a consequence of (A1.3).

Substituting (A1.3) into (A1.6) gives

$$\nabla\left(\dot{N}^\alpha + \mathbf{a}^\alpha \dot{\mathbf{u}}\right) = 0$$

and hence,

$$\dot{\mathbf{u}} = -\mathbf{a}_\alpha \dot{N}^\alpha, \quad \dot{N}^\alpha = -\dot{\mathbf{u}}\mathbf{a}^\alpha. \qquad (A1.7)$$

It follows from (A1.3) and (A1.7) that

$$dN^\alpha = \frac{\partial N^\alpha}{\partial \mathbf{r}} d\mathbf{r} + \frac{\partial N^\alpha}{\partial t} dt = \mathbf{a}^\alpha d\mathbf{r} - \mathbf{a}^\alpha \dot{\mathbf{u}} dt \qquad (A1.8)$$

This expression coincides with that given in Notation. It can be written also in the form used in the text:

$$d\mathbf{r} = \mathbf{a}_\alpha dN^\alpha + \dot{\mathbf{u}} dt$$

APPENDIX 2

To obtain the time derivarive of the quasiparticle energy density one can use the kinetic equation (44) for $\varphi(\boldsymbol{\kappa}, \mathbf{r}, t)$. Since the fluxes through the Brillouin zone boundaries in this notation equal zero it can be written in its integrated by parts form

$$\dot{\varphi} + \mathrm{div}\left\{\left(\dot{\mathbf{u}} + \mathbf{a}_\alpha \frac{\partial \epsilon}{\partial \kappa_\alpha}\right)\varphi\right\} + \mathbf{a}_\alpha \left\{\nabla\epsilon + m\frac{\mathrm{d}\dot{\mathbf{u}}}{\mathrm{d}t} - m\frac{\partial \epsilon}{\partial \kappa_\beta}\mathbf{a}_\beta \times \mathrm{curl}\,\dot{\mathbf{u}} - \mathbf{F}\right\}\varphi \frac{\partial}{\partial \kappa_\alpha} = \hat{I}\varphi. \qquad (A2.1)$$

We have to transform the expresssion

$$\frac{\partial}{\partial t}\langle \epsilon f\rangle = \langle\langle \dot{\epsilon}\varphi\rangle\rangle + \langle\langle \epsilon\dot{\varphi}\rangle\rangle. \qquad (A2.2)$$

As the derivative with respect to t is taken at constant $\boldsymbol{\kappa}$ one can use the same procedure, as when obtaining (61). This yields

$$\dot{\epsilon} = \left(\frac{\partial \epsilon}{\partial t}\right)_{\boldsymbol{\kappa}} = \frac{1}{2}\lambda_{\alpha\beta}\dot{g}^{\alpha\beta} = -\lambda_{\alpha\beta}a_i^\alpha a_k^\beta \frac{\partial \dot{u}_i}{\partial x_k} - \dot{u}(\nabla\epsilon)_{\boldsymbol{\kappa}}\,, \qquad (A2.3)$$

+ where

$$\lambda_{\alpha\beta} = 2\frac{\partial \epsilon}{\partial g^{\alpha\beta}} = \lambda_{\beta\alpha} \qquad (A2.4)$$

Multiplying (A2.3) by φ and (A2.1) by ϵ one obtains after substituting into (A2.2):

$$\begin{aligned}
\frac{\partial}{\partial t}\langle \epsilon f\rangle &= \langle\langle \epsilon \hat{I}\varphi\rangle\rangle - \mathrm{div}\left(\dot{\mathbf{u}}\langle\langle\epsilon\varphi\rangle\rangle + \mathbf{a}_{\alpha\beta}\left\langle\left\langle \epsilon\frac{\partial\epsilon}{\partial k_\alpha}\varphi\right\rangle\right\rangle\right) + \mathbf{a}_\alpha\left\langle\left\langle \mathbf{F}\frac{\partial\epsilon}{\partial k_\alpha}\varphi\right\rangle\right\rangle \\
&- m\ddot{\mathbf{u}}\mathbf{a}_\alpha\left\langle\left\langle\frac{\partial\epsilon}{\partial k_\alpha}\varphi\right\rangle\right\rangle - \frac{\partial u_i}{\partial x_k}\left(m\dot{u}_k a_{\alpha i}\left\langle\left\langle\frac{\partial\epsilon}{\partial k_\alpha}\varphi\right\rangle\right\rangle + a_i^\alpha a_k^\beta\langle\langle\lambda_{\alpha\beta}\varphi\rangle\rangle\right) \\
&= \langle\epsilon\hat{I}f\rangle - \mathrm{div}\left(\dot{\mathbf{u}}\langle\epsilon f\rangle + \left\langle\epsilon\frac{\partial\epsilon}{\partial\mathbf{p}}f\right\rangle\right) + \left\langle\mathbf{F}\frac{\partial\epsilon}{\partial\mathbf{p}}f\right\rangle - m\ddot{\mathbf{u}}\left\langle\frac{\partial\epsilon}{\partial\mathbf{p}}f\right\rangle \\
&- \frac{\partial\dot{u}_i}{\partial x_k}\left(m\dot{u}_k\left\langle\frac{\partial\epsilon}{\partial p_i}f\right\rangle + a_i^\alpha a_k^\beta\langle\lambda_{\alpha\beta}f\rangle\right)
\end{aligned}$$

where the rules (45) are used to replace the double brakets by single ones. This expression is used when obtaining (64).

We would like here to pointed out how easily this formula has been obtained. For comparisson, the expression which corresponds to (A2.2) in variables $\mathbf{p}, \mathbf{r}, t$ has the form:

$$\left(\frac{\partial\epsilon}{\partial t}\right)_{\mathbf{p}} = -\lambda_{\alpha\beta}a_i^\alpha a_k^\beta\frac{\partial\dot{u}_i}{\partial x_k} - m\ddot{\mathbf{u}}\frac{\partial\epsilon}{\partial\mathbf{p}} - \dot{\mathbf{u}}(\nabla\epsilon)_{\mathbf{p}} + \left(p_i\frac{\partial\epsilon}{\partial p_k} - m\ddot{u}_k\right)\frac{\partial\dot{u}_i}{\partial x_k}.$$

This expression is both cumbersom and nonperiodic which creates additional difficulties.

APPENDIX 3

The lattice contribution L_{ik} to the momentum flux density tensor Π_{ik} is given by (68). The term in the brakets can be written in the form:

$$\sigma_{ik} + E_0\delta_{ik} = \left(\sigma_{\alpha\beta} + E_0 g_{\alpha\beta}\right)a_i^\alpha a_k^\beta \qquad (A3.1)$$

where $E_0(g^{\alpha\beta})$ is the strain energy per unit volume.

In an isotropic medium the dependence of E_0 on $g^{\alpha\beta}$ is reduced to a dependence on $g = \det g_{\alpha\beta}$:

$$\sigma_{\alpha\beta} = -2\frac{\partial E_0}{\partial g}\frac{\partial g}{\partial g^{\alpha\beta}}.$$

By the well known formula

$$dg = -g g_{\alpha\beta} dg^{\alpha\beta}$$

one obtains easily

$$\sigma_{\alpha\beta} = 2g\frac{\partial E_0}{\partial g} a_{\alpha l} a_{\beta l}$$

and hence

$$\sigma_{ik} = 2g\frac{\partial E_0}{\partial g} a_{\alpha l} a_{\beta l} a_i^\alpha a_k^\beta = 2g\frac{\partial E_0}{\partial g}\delta_{ik} \qquad (A3.2)$$

Taking into account that g is the squared volume of a unit cell ($g = V^2$) and that $V = 1/\rho$ one obtains

$$\sigma_{ik} = -\rho\frac{\partial E_0}{\partial \rho}\delta_{ik} \qquad (A3.3)$$

Hence, it is seen, that in an isotropic case

$$2a_i^\alpha a_k^\beta \frac{\partial}{\partial g^{\alpha\beta}} \quad \rightarrow \quad \delta_{ik}\rho\frac{\partial}{\partial \rho}. \qquad (A3.4)$$

Let now $\tilde{E}_0$ and s be the internal energy and entropy per unit mass ($\tilde{E}_0 = E_0 V$). Making use of the thermodynamic relation

$$d\tilde{E}_0 = Tds - PdV + \mu dN \qquad (A3.5)$$

one can define the pressure P as

$$P = -\left(\frac{\partial \tilde{E}_0}{\partial V}\right)_{s,N} = -\left(\frac{\partial E_0 V}{\partial V}\right)_{s,N} = -E_0 + \rho\left(\frac{\partial E_0}{\partial \rho}\right)_{s,N} \qquad (A3.6)$$

It follows from (A3.3) and (A3.6) that in isotropic medium

$$\sigma_{ik} + E_0\delta_{ik} = -P\delta_{ik} \qquad (A3.7)$$

It is supposed in our consideration that the only contribution to the entropy is due to quasiparticles and this contribution comes from the kinetic equation. Therefore, the

derivatives of E_0 with respect to the metrical tensor components $g^{\alpha\beta}$ are assumed as taken at constant entropy.

An alternative approach can be based on the free energy thermodynamic potential per unit volume $F(T, P, N)$. In that case one obtains

$$-P = F - \rho \left(\frac{\partial F}{\partial \rho} \right)_T \qquad (A3.8)$$

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